Amplifier DC Analysis
by
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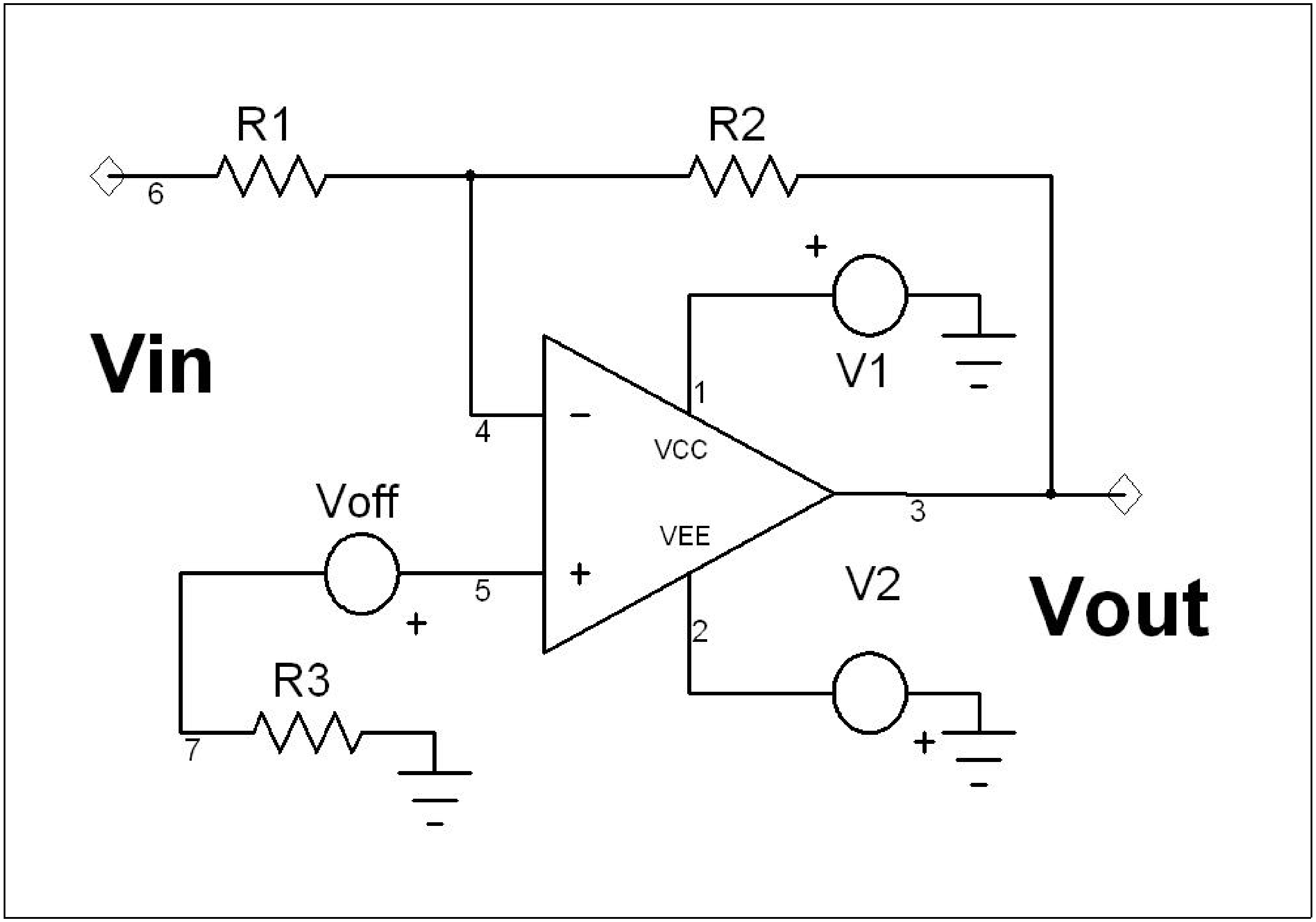
Part A: Inverting Amplifier Case



Amplifier DC performance is affected by a variety of Op Amp characteristics. Not all of these factors are commonly well understood. This analysis will develop complete expressions for analyzing the DC performance of an inverting amplifier including the effect of bias current, offset current and offset voltage. Accurate assessment of performance over a wide range of conditions will then be possible. The two amplifier input terminal currents will be labeled I_{minus} and I_{plus} . Most bipolar Op Amps (but not all) have input stages arranged such that current flows into the amplifier input pins. The situation is more complex with FET input devices where the dominant input current term is a leakage current.

By superposition:

$$V_4 = V_{in} \cdot \frac{R_2}{R_1 + R_2} + V_{out} \cdot \frac{R_1}{R_1 + R_2} - I_{minus} \cdot \left(\frac{1}{\frac{1}{R_1} + \frac{1}{R_2}} \right)$$

EQ. 1

I_{minus} will be defined as positive when it flows into the Op Amp negative input terminal.

$$V_5 = -I_{plus} \cdot R_3 + V_{off}$$

EQ. 2

I_{plus} will be defined as positive when it flows into the Op Amp positive input terminal. The positive end of Voff is indicated in the diagram

$$V_{out} = -(V_4 - V_5) \cdot G$$

EQ. 3

G is the open loop gain of the Op Amp and can be broken down further into a DC term multiplied by an expression accounting for gain changes with frequency.

$$I_{bias} = \frac{I_{plus} + I_{minus}}{2}$$

$$I_{off} = I_{minus} - I_{plus}$$

We will define offset current to be positive when it results from a current (I_{off}/2) that flows into the Op Amp negative input terminal and out of the positive input terminal.

$$H = \frac{R_1}{R_1 + R_2}$$

Therefore

$$1 - H = \frac{R_2}{R_1 + R_2}$$

H is (by definition) the gain from V_{out} to V_4 with all other voltage sources shorted and all current sources open (Op Amp loading of V_4 ignored). This is part of the analysis by the very powerful superposition process.

In view of the above:

$$V_4 = V_{in} \cdot (1 - H) + V_{out} \cdot (H) - \left(I_{bias} + \frac{I_{off}}{2} \right) \cdot \left(\frac{1}{\frac{1}{R_1} + \frac{1}{R_2}} \right)$$

EQ. 4

Effective voltage at the Op Amp inverting input after the substitution of expressions involving H, I_{bias} and I_{off}.

$$V_5 = -\left(I_{bias} - \frac{I_{off}}{2} \right) \cdot R_3 + V_{off}$$

EQ. 5

Effective voltage at the Op Amp non-inverting input after the substitution of expressions involving I_{bias} and I_{off}.

$$V_{out} = -\left[V_{in}\cdot(1-H) + V_{out}\cdot(H) - \left(I_{bias} + \frac{I_{off}}{2}\right)\cdot\left(\frac{1}{\frac{1}{R_1}+\frac{1}{R_2}}\right) - \left[-\left(I_{bias} - \frac{I_{off}}{2}\right)\cdot R_3 + V_{off}\right]\right]\cdot G$$

Substituting EQ. 4 and EQ. 5 into EQ. 3

$$V_{out} = -\left[V_{in}\cdot(1-H) + V_{out}\cdot(H) - I_{bias}\cdot\left[\left(\frac{1}{\frac{1}{R_1}+\frac{1}{R_2}}\right) - R_3\right] - \frac{I_{off}}{2}\cdot\left[\left(\frac{1}{\frac{1}{R_1}+\frac{1}{R_2}}\right) + R_3\right] - V_{off}\right]\cdot G$$

Collecting terms involving I_{bias} and I_{off}.

$$V_{out} = -\left[V_{in}\cdot(1-H) - I_{bias}\cdot\left[\left(\frac{1}{\frac{1}{R_1}+\frac{1}{R_2}}\right) - R_3\right] - \frac{I_{off}}{2}\cdot\left[\left(\frac{1}{\frac{1}{R_1}+\frac{1}{R_2}}\right) + R_3\right] - V_{off}\right]\cdot\frac{G}{1+G\cdot H}$$

Collecting terms involving V_{out}.

Next we will put the expression in a particular version of standard form that is a very low entropy expression that brings out the relationships that drive performance.

$$V_{out} = -\left(\frac{1-H}{H}\right)\cdot\left(\frac{G\cdot H}{1+G\cdot H}\right)\cdot V_{in} + \left(\frac{1}{H}\right)\cdot\left(\frac{G\cdot H}{1+G\cdot H}\right)\cdot\left[\left(\frac{1}{\frac{1}{R_1}+\frac{1}{R_2}}\right) - R_3\right]\cdot I_{bias} + \left(\frac{1}{H}\right)\cdot\left(\frac{G\cdot H}{1+G\cdot H}\right)\cdot\left[\left(\frac{1}{\frac{1}{R_1}+\frac{1}{R_2}}\right) + R_3\right]\cdot\frac{I_{off}}{2} + \frac{1}{H}\cdot\left(\frac{G\cdot H}{1+G\cdot H}\right)\cdot V_{off}$$

EQ. 6

Examination of this last expression reveals that the output is expressed as the sum of several terms. The first of these terms is a function of the actual desired input signal and the rest are functions of undesired, but very real, parasitic inputs (input voltages and currents that are generated by the Op Amp). Each of the effective inputs (except the actual intended input signal) has been arranged so that the gain expression is recognized as being in the standard non-inverting format. Only the actual intended input signal is subjected to the inverting gain function (-(1-H)/H). Further examination of this expression reveals that each term on the right is composed of an expression that assumes infinite Op Amp gain times a correction term (G*H/(1+G*H)) that accounts for Op Amp gain effects. The two terms involving currents also include an equivalent resistance to convert the current to a voltage. Closer examination of these resistance terms reveals that the coefficient of I_{bias} is the difference between two resistances while the coefficient of I_{off} is the sum of these two resistance terms. The term involving R_1 and R_2 is the parallel combination of these two resistances. One can quickly recognize this term as the Thevenin equivalent resistance attached to the Op Amp inverting input. An excellent thing happens when the value of R_3 is set equal to the Thevenin equivalent impedance at the inverting input. The term involving I_{bias} is now multiplied by zero and disappears. Then the error term involving the currents is reduced to one involving only I_{off} which is usually much smaller than I_{bias}. This explains the reason for matching the impedances at the two Op Amp inputs. Designers also frequently try to match the AC impedances (at the amplifier inputs) across frequency although the benefit gets much smaller very quickly as the amplifier gain decreases and the differential input current grows.

Taking the derivatives of the left hand side of EQ. 6 with respect to the individual input signals reveals the corresponding gain terms:

$$\frac{\Delta V_{out}}{\Delta V_{in}} = -\left(\frac{1-H}{H}\right)\cdot\left(\frac{G\cdot H}{1+G\cdot H}\right)$$

The classic inverting amplifier gain expression. G*H is the loop gain which is the open loop gain of the Op Amp times the transfer function of the feedback path from output to inverting input.

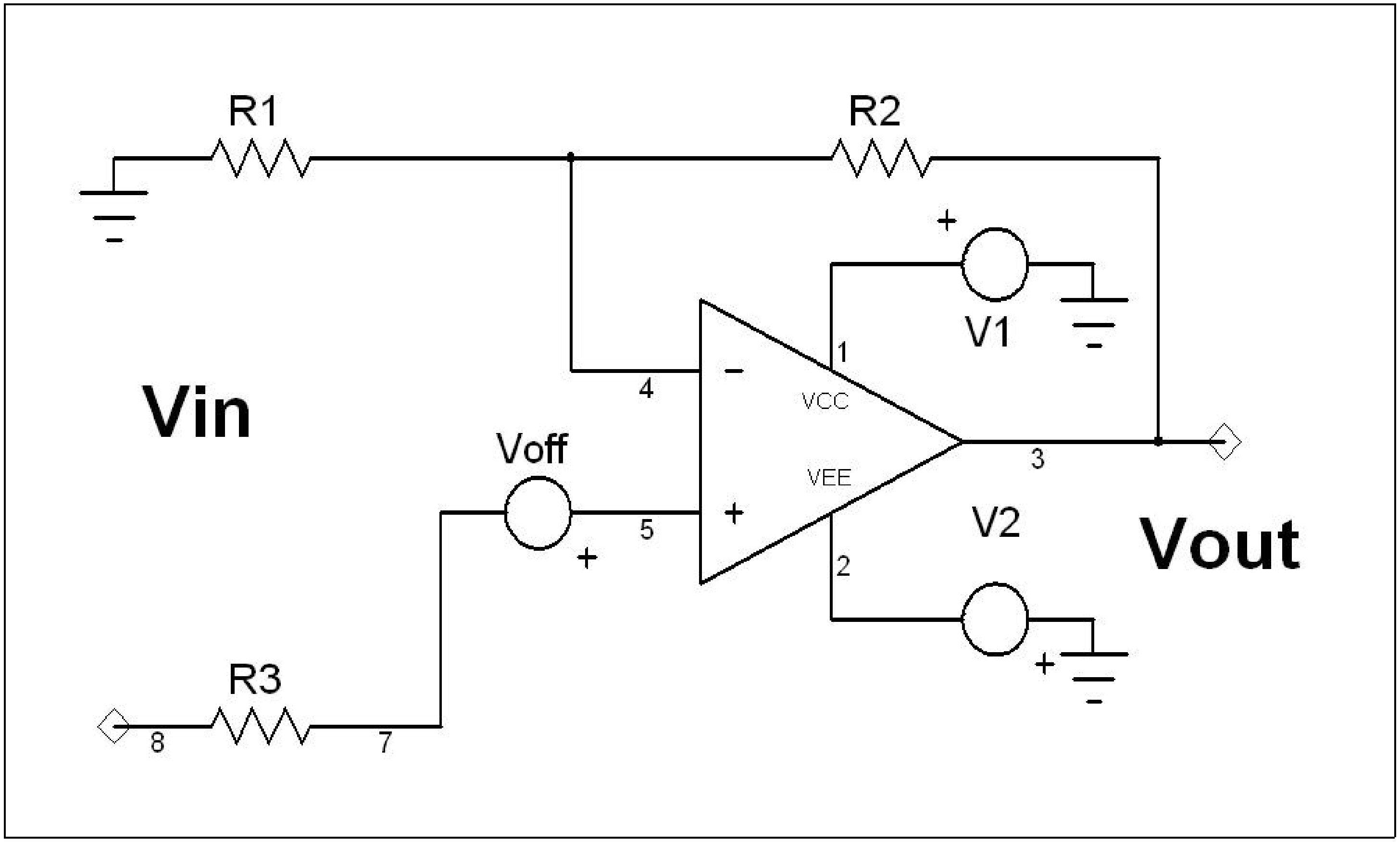
$$\frac{\Delta V_{out}}{\Delta\left[\left(\frac{1}{\frac{1}{R_1}+\frac{1}{R_2}}\right) - R_3\right]\cdot I_{bias}} = \left(\frac{1}{H}\right)\cdot\left(\frac{G\cdot H}{1+G\cdot H}\right)$$

The classic non-inverting amplifier gain expression. The other two gain terms will have this same form.

This analysis has developed a complete expression for the DC gain of the amplifier including the effects of Amplifier Gain, I_{bias}, I_{off} and V_{off}. This expression can now be used to predict output voltage stability. For example, if the loop gain is 1,000 then G*H/(1+G*H)=1,000/1,001~0.999 so the ideal gain is degraded by 0.1% error due to low loop gain. A typical Op Amp has a DC gain that is very high so H would have to be a very small number (big attenuator) to cause this much gain error. In a similar fashion, output voltage error terms can be computed for I_{bias}, I_{off} and V_{off}. Amplifier specifications provide expected variations in all these parameters over temperature so the effect of these variations can easily determined. The expression for G*H does not include the DC signal inversion (internal to the Op Amp) represented by the inverting input pin. Therefore the value of G is a positive number at DC in spite of the minus sign on the inverting input pin. This signal inversion is accounted for by the minus sign just after the equals sign in EQ. 6.

It should be noted that amplifier input and output voltages must remain within their specified linear ranges for the above analysis to hold. Most amplifiers do not allow rail-to-rail operation of input and output voltages. It should also be noted that Op Amps have a specified minimum input impedance (common mode and differential mode). This means that Op Amp input currents are more complex than simple current sources and will be somewhat affected by input voltage variations. An even more accurate model would result if these impedances were included in the gain expression. This effect becomes more important as the impedances external to the Op Amp become larger and does not significantly affect most applications.

Part B: Non-Inverting Amplifier Case



Proceeding as before:

By superposition:

$$V_4 = V_{out} \cdot \frac{R_1}{R_1 + R_2} - I_{minus} \cdot \left(\frac{1}{\frac{1}{R_1} + \frac{1}{R_2}} \right)$$

EQ. 7

I_{minus} will be defined as positive when it flows into the Op Amp negative input terminal.

$$V_5 = V_{in} - I_{plus} \cdot R_3 + V_{off}$$

EQ. 8

I_{plus} will be defined as positive when it flows into the Op Amp positive input terminal. The positive end of Voff is indicated in the diagram

$$V_{out} = (V_5 - V_4) \cdot G$$

EQ. 9

G is the open loop gain of the Op Amp and can be broken down further into a DC term multiplied by an expression accounting for gain changes with frequency.

$$I_{bias} = \frac{I_{plus} + I_{minus}}{2} \qquad I_{off} = I_{minus} - I_{plus}$$

We will define offset current to be positive when it results from a current ($I_{off}/2$) that flows into the Op Amp negative input terminal and out of the positive input terminal.

$$H = \frac{R_1}{R_1 + R_2} \quad \text{Therefore} \quad \frac{1}{H} = \frac{R_1 + R_2}{R_1}$$

H is (by definition) the gain from V_{out} to V_4 with all other voltage sources shorted and all current sources open (Op Amp loading of V_4 ignored). This is part of the analysis by the very powerful superposition process.

In view of the above:

$$V_4 = V_{out} \cdot H - \left(I_{bias} + \frac{I_{off}}{2} \right) \cdot \left(\frac{1}{\frac{1}{R_1} + \frac{1}{R_2}} \right)$$

EQ. 10

Effective voltage at the Op Amp inverting input after the substitution of expressions involving H, I_{bias} and I_{off}.

$$V_5 = V_{in} - \left(I_{bias} - \frac{I_{off}}{2} \right) \cdot R_3 + V_{off}$$

EQ. 11

Effective voltage at the Op Amp non-inverting input after the substitution of expressions involving I_{bias} and I_{off}.

$$V_{out} = \left[V_{in} - \left(I_{bias} - \frac{I_{off}}{2} \right) \cdot R_3 + V_{off} - \left[V_{out} \cdot H - \left(I_{bias} + \frac{I_{off}}{2} \right) \cdot \left(\frac{1}{\frac{1}{R_1} + \frac{1}{R_2}} \right) \right] \right] \cdot G$$

Substituting EQ. 10 and EQ. 11 into EQ. 9

$$V_{out} = \left[V_{in} - V_{out}\cdot(H) + I_{bias}\cdot\left[\left(\frac{1}{\frac{1}{R_1}+\frac{1}{R_2}}\right) - R_3\right] + \frac{I_{off}}{2}\cdot\left[\left(\frac{1}{\frac{1}{R_1}+\frac{1}{R_2}}\right) + R_3\right] + V_{off}\right]\cdot G$$

Collecting terms involving I_{bias} and I_{off}.

$$V_{out} = \left[V_{in} + I_{bias}\cdot\left[\left(\frac{1}{\frac{1}{R_1}+\frac{1}{R_2}}\right) - R_3\right] + \frac{I_{off}}{2}\cdot\left[\left(\frac{1}{\frac{1}{R_1}+\frac{1}{R_2}}\right) + R_3\right] + V_{off}\right]\cdot\frac{G}{1+G\cdot H}$$

Collecting terms involving V_{out}.

Next we will put the expression in a particular version of standard form that is a very low entropy expression that brings out the relationships that drive performance.

$$V_{out} = \left(\frac{1}{H}\right)\cdot\left(\frac{G\cdot H}{1+G\cdot H}\right)\cdot V_{in} + \left(\frac{1}{H}\right)\cdot\left(\frac{G\cdot H}{1+G\cdot H}\right)\cdot\left[\left(\frac{1}{\frac{1}{R_1}+\frac{1}{R_2}}\right) - R_3\right]\cdot I_{bias} + \left(\frac{1}{H}\right)\cdot\left(\frac{G\cdot H}{1+G\cdot H}\right)\cdot\left[\left(\frac{1}{\frac{1}{R_1}+\frac{1}{R_2}}\right) + R_3\right]\cdot\frac{I_{off}}{2} + \frac{1}{H}\cdot\left(\frac{G\cdot H}{1+G\cdot H}\right)\cdot V_{off}$$

EQ. 12

As before, examination of this last expression reveals that the output is expressed as the sum of several terms. The first of these terms is a function of the actual desired input signal and the rest are functions of undesired, but very real, parasitic inputs (input voltages and currents that are generated by the Op Amp). Each of the effective inputs has been arranged so that the gain expression is recognized as being in the standard non-inverting format. Further examination of this expression reveals that each term on the right is composed of an expression that assumes infinite Op Amp gain times a correction term (G*H/(1+G*H)) that accounts for Op Amp gain effects. The two terms involving currents also include an equivalent resistance to convert the current to a voltage. Closer examination of these resistance terms reveals that the coefficient of I_{bias} is the difference between two resistances while the coefficient of I_{off} is the sum of these two resistance terms. The term involving R_1 and R_2 is the parallel combination of these two resistances. One can quickly recognize this term as the Thevenin equivalent resistance attached to the Op Amp inverting input. An excellent thing happens when the value of R_3 is set equal to the Thevenin equivalent impedance at the inverting input. The term involving I_{bias} is now multiplied by zero and disappears. Then the error term involving the currents is reduced to one involving only I_{off} which is usually much smaller than I_{bias}. This explains the reason for matching the impedances at the two Op Amp inputs. Designers also frequently try to match the AC impedances (at the amplifier inputs) across frequency although the benefit gets much smaller very quickly as the amplifier gain decreases and the differential input current grows.

Taking the derivatives of the left hand side of EQ. 12 with respect to the individual input signals reveals the corresponding gain terms:

$$\frac{\Delta V_{out}}{\Delta V_{in}} = \left(\frac{1}{H}\right)\cdot\left(\frac{G\cdot H}{1 + G\cdot H}\right)$$

The classic non-inverting amplifier gain expression. G*H is the loop gain which is the open loop gain of the Op Amp times the transfer function of the feedback path from output to inverting input.

$$\frac{\Delta V_{out}}{\Delta\left[\left(\frac{1}{\frac{1}{R_1} + \frac{1}{R_2}}\right) - R_3\right]\cdot I_{bias}} = \left(\frac{1}{H}\right)\cdot\left(\frac{G\cdot H}{1 + G\cdot H}\right)$$

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It should be noted that amplifier input and output voltages must remain within their specified linear ranges for the above analysis to hold. Most amplifiers do not allow rail-to-rail operation of input and output voltages (although some of the newer ones do). It should also be noted that Op Amps have a specified minimum input impedance (common mode and differential mode). This means that Op Amp input currents are more complex than simple current sources and will be somewhat affected by input voltage variations. An even more accurate model would result if these impedances were included in the gain expression. This effect becomes more important as the impedances external to the Op Amp become larger and does not significantly affect most applications.