Amplifier AC Design Problem
by
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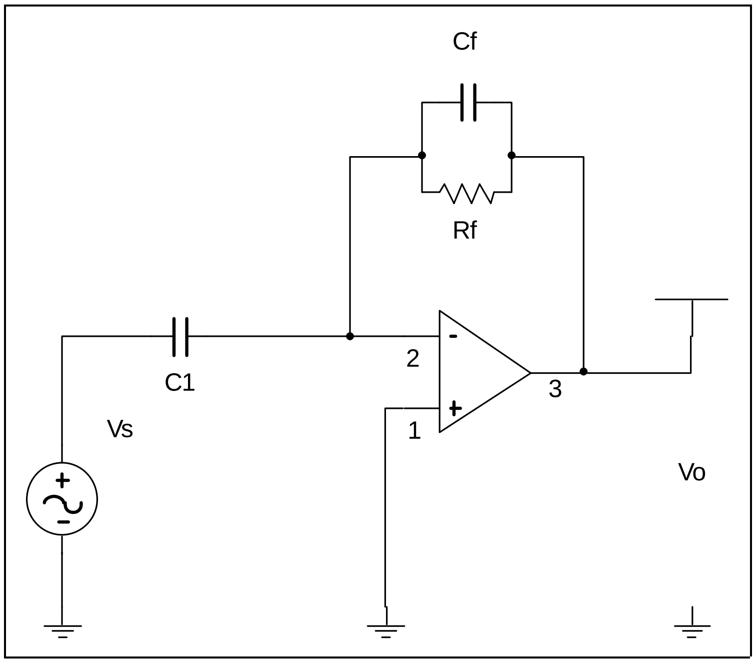
Here is the analysis of an amplifier that shows the development of the gain expressions for an inverting amplifier and illustrates the origin of the difficulty in getting the amplifier to work as desired. This hypothetical amplifier was intended to be a high pass filter with the gain breaking flat (+60dB) at ~ 160 Hz and the maximum frequency of interest being 5kHz. The amplifier does not work as intended and this analysis will show why. This analysis is somewhat simplified in that it ignores Op Amp bias and offset currents and offset voltage since DC performance was not the object of the analysis. Real Op Amps also exhibit additional break frequencies not modeled here (usually near open loop gain cross-over) but again these were irrelevant to this analysis. Parasitic reactances associated with the layout of PC Boards can also affect the high frequency performance of real amplifiers.

$R_f := 50 \cdot 10^{3}$ $\quad C_1 := 20 \cdot 10^{-6}$ $\quad G_0 := 1 \cdot 10^{5}$ $\quad C_f := 0.02 \cdot 10^{-6}$



By superposition:

$$V_2 = \left(\frac{Z_{C1}}{\frac{1}{\frac{1}{Z_{Cf}} + \frac{1}{Rf}} + Z_{C1}}\right) \cdot V_o + \left[\frac{\frac{1}{\frac{1}{Z_{Cf}} + \frac{1}{Rf}}}{\left(\frac{1}{\frac{1}{Z_{Cf}} + \frac{1}{Rf}} + Z_{C1}\right)}\right] \cdot V_s$$

Eq. 1

By definition:

$$V_o = -V_2 \cdot G$$

where G is the open loop gain of the amplifier

Therefore:

$$V_2 = \frac{-V_o}{G}$$

Eq. 2

If we examine Eq. 1, we can define the coefficient of V_o as H. Then the coefficient of V_s will be 1-H and we can combine Eq. 1 with Eq. 2 and rewrite the expression as follows:

$$\frac{-V_o}{G} = H \cdot V_o + (1 - H) \cdot V_s$$

$$V_o \cdot \left(\frac{1 + G \cdot H}{G}\right) = -\left[(1 - H) \cdot V_s\right]$$

Collecting terms involving V_o

$$\frac{V_o}{V_s} = -\left(\frac{1 - H}{H}\right) \cdot \left(\frac{G \cdot H}{1 + G \cdot H}\right)$$

Solving for circuit gain

Eq. 3

This last expression consists of an ideal gain term -(1-H)/H time a correction factor (G*H/(1+G*H)) that accounts for the non-ideal properties of the Op Amp.

$i := 0, 0.1 .. 4$ $\quad f(i) := 10^{i}$ Here we set up a range variable that allows us to sweep frequency.

$\omega(i) := 2 \cdot \pi \cdot f(i)$ $\quad G(i) := \dfrac{G_0}{1 + j \cdot 0.1 \cdot \omega(i)}$

$S(i) := j \cdot \omega(i)$ in the sinusoidal steady state

$$H(i) := \frac{\dfrac{1}{S(i) \cdot C_1}}{\left(\dfrac{1}{S(i) \cdot C_f + \dfrac{1}{R_f}} + \dfrac{1}{S(i) \cdot C_1} \right)}$$

$$1 - H(i) = \left[\frac{\dfrac{1}{S(i) \cdot C_f + \dfrac{1}{R_f}}}{\left(\dfrac{1}{S(i) \cdot C_f + \dfrac{1}{R_f}} + \dfrac{1}{S(i) \cdot C_1} \right)} \right]$$

$$G_{CL} = \frac{V_o}{V_s}$$

$$-\left(\frac{1 - H(i)}{H(i)} \right) = -\left(\frac{\dfrac{1}{S(i) \cdot C_f + \dfrac{1}{R_f}}}{\dfrac{1}{S(i) \cdot C_1}} \right)$$

$$G_{CL}(i) := -\left(\frac{1 - H(i)}{H(i)} \right) \cdot \left(\frac{G(i) \cdot H(i)}{1 + G(i) \cdot H(i)} \right)$$

Eq. 4

This is the closed loop gain of the amplifier. This expression determines the gain and phase shift applied to signals processed by this amplifier.

Now we will assemble expressions for the magnitudes (dB) of actual of amplifier closed loop gain ($G_{CL\_MAG}(i)$), ideal amplifier closed loop gain ($G_{CL1\_MAG}(i)$) assuming a perfect Op Amp and the open loop gain ($G_{OL\_MAG}(i)$) of the chosen Op Amp. We will plot the results.

$$G_{CL\_MAG}(i) := 20 \cdot \log\left(\left|G_{CL}(i)\right|\right)$$

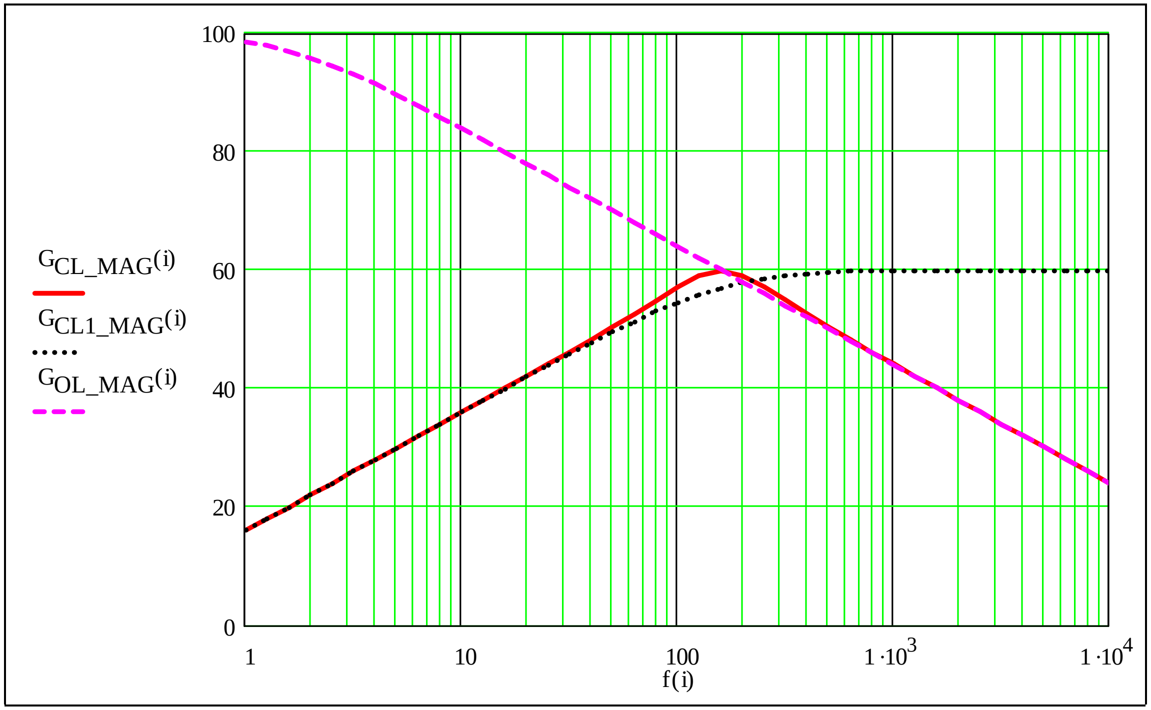
Eq. 5

$$G_{CL1\_MAG}(i) := 20 \cdot \log\left[\left|-\left(\frac{1 - H(i)}{H(i)}\right)\right|\right]$$

Eq. 6

$$G_{OL\_MAG}(i) := 20 \cdot \log\left(\left|\frac{G_0}{1 + j \cdot 0.1 \cdot \omega(i)}\right|\right)$$

Eq. 7



Here we can see the interplay of amplifier open loop gain ($G_{OL\_MAG}(i)$) and ideal closed loop gain ($G_{CL1\_MAG}(i)$) resulting in actual closed loop gain ($G_{CL\_MAG}(i)$). The amplifier runs out of gain and fails to follow the ideal gain expression above ~ 100Hz.